

Table 1

Exclusion Criteria for Estimating the Controlled Direct Effects with a Binary Mediator

Selected Subpopulations	Exclusion Criteria
Always Employed under Either LFA or Control	$\theta_{Z_1} > \text{Max}(\theta_{Z_1}   A = 1, Z = 0)$ or $\theta_{Z_0} > \text{Max}(\theta_{Z_0}   A = 0, Z = 0)$
Always Employed under Either LFA or Control	$\theta_{Z_1} < \text{Min}(\theta_{Z_1}   A = 1, Z = 1)$ or $\theta_{Z_0} < \text{Min}(\theta_{Z_0}   A = 0, Z = 1)$

Table 2

## Inclusion Criteria for Estimating the Natural Direct and Indirect Effects with a Binary Mediator

Selected Subpopulations	Inclusion Criteria
Always Employed Regardless of Treatment	$\theta_{Z_1} > \text{Max}(\theta_{Z_1}   A = 1, Z = 0)$ and $\theta_{Z_0} > \text{Max}(\theta_{Z_0}   A = 0, Z = 0)$
Never Employed Regardless of Treatment	$\theta_{Z_1} < \text{Min}(\theta_{Z_1}   A = 1, Z = 1)$ and $\theta_{Z_0} < \text{Min}(\theta_{Z_0}   A = 0, Z = 1)$
Probably Employed under LFA and Control	$\text{Min}(\theta_{Z_1}   A = 1, Z = 1) \leq \theta_{Z_1} \leq \text{Max}(\theta_{Z_1}   A = 1, Z = 0)$ and $\text{Min}(\theta_{Z_0}   A = 0, Z = 1) \leq \theta_{Z_0} \leq \text{Max}(\theta_{Z_0}   A = 0, Z = 0)$

Table 3

## Exclusion Criteria for Estimating the Controlled Direct Effects with a Three-Category Mediator

Selected Subpopulations	Exclusion Criteria
Never Having Zero or Low or High Employment if Treated	$\theta_{Z_1=0} > \text{Min}[\text{Max}(\theta_{Z_1=0} A = 1, Z = 0), \text{Max}(\theta_{Z_1=0} A = 1, Z = 1), \text{Max}(\theta_{Z_1=0} A = 1, Z = 2)]$
	$\theta_{Z_1=0} < \text{Max}[\text{Min}(\theta_{Z_1=0} A = 1, Z = 0), \text{Min}(\theta_{Z_1=0} A = 1, Z = 1), \text{Min}(\theta_{Z_1=0} A = 1, Z = 2)]$
	$\theta_{Z_1=1} > \text{Min}[\text{Max}(\theta_{Z_1=1} A = 1, Z = 0), \text{Max}(\theta_{Z_1=1} A = 1, Z = 1), \text{Max}(\theta_{Z_1=1} A = 1, Z = 2)]$
	$\theta_{Z_1=1} < \text{Max}[\text{Min}(\theta_{Z_1=1} A = 1, Z = 0), \text{Min}(\theta_{Z_1=1} A = 1, Z = 1), \text{Min}(\theta_{Z_1=1} A = 1, Z = 2)]$
	$\theta_{Z_1=2} > \text{Min}[\text{Max}(\theta_{Z_1=2} A = 1, Z = 0), \text{Max}(\theta_{Z_1=2} A = 1, Z = 1), \text{Max}(\theta_{Z_1=2} A = 1, Z = 2)]$
	$\theta_{Z_1=2} < \text{Max}[\text{Min}(\theta_{Z_1=2} A = 1, Z = 0), \text{Min}(\theta_{Z_1=2} A = 1, Z = 1), \text{Min}(\theta_{Z_1=2} A = 1, Z = 2)]$
Never Having Zero or Low or High Employment if Untreated	$\theta_{Z_0=0} > \text{Min}[\text{Max}(\theta_{Z_0=0} A = 0, Z = 0), \text{Max}(\theta_{Z_0=0} A = 0, Z = 1), \text{Max}(\theta_{Z_0=0} A = 0, Z = 2)]$
	$\theta_{Z_0=0} < \text{Max}[\text{Min}(\theta_{Z_0=0} A = 0, Z = 0), \text{Min}(\theta_{Z_0=0} A = 0, Z = 1), \text{Min}(\theta_{Z_0=0} A = 0, Z = 2)]$
	$\theta_{Z_0=1} > \text{Min}[\text{Max}(\theta_{Z_0=1} A = 0, Z = 0), \text{Max}(\theta_{Z_0=1} A = 0, Z = 1), \text{Max}(\theta_{Z_0=1} A = 0, Z = 2)]$
	$\theta_{Z_0=1} < \text{Max}[\text{Min}(\theta_{Z_0=1} A = 0, Z = 0), \text{Min}(\theta_{Z_0=1} A = 0, Z = 1), \text{Min}(\theta_{Z_0=1} A = 0, Z = 2)]$
	$\theta_{Z_0=2} > \text{Min}[\text{Max}(\theta_{Z_0=2} A = 0, Z = 0), \text{Max}(\theta_{Z_0=2} A = 0, Z = 1), \text{Max}(\theta_{Z_0=2} A = 0, Z = 2)]$
	$\theta_{Z_0=2} < \text{Max}[\text{Min}(\theta_{Z_0=2} A = 0, Z = 0), \text{Min}(\theta_{Z_0=2} A = 0, Z = 1), \text{Min}(\theta_{Z_0=2} A = 0, Z = 2)]$

Table 4

## Inclusion Criteria for Estimating the Natural Direct Effect and Natural Indirect Effect with a Three-Category Mediator

Selected Subpopulations	Inclusion Criteria
Always Having Zero Employment Regardless of Treatment	$\theta_{Z_1=0} > \text{Max}[\text{Max}(\theta_{Z_1=0} A = 1, Z = 1), \text{Max}(\theta_{Z_1=0} A = 1, Z = 2)]$ and $\theta_{Z_0=0} > \text{Max}[\text{Max}(\theta_{Z_0=0} A = 0, Z = 1), \text{Max}(\theta_{Z_0=0} A = 0, Z = 2)]$
Always Having Low Employment Regardless of Treatment	$\theta_{Z_1=1} > \text{Max}[\text{Max}(\theta_{Z_1=1} A = 1, Z = 0), \text{Max}(\theta_{Z_1=1} A = 1, Z = 2)]$ and $\theta_{Z_0=1} > \text{Max}[\text{Max}(\theta_{Z_0=1} A = 0, Z = 0), \text{Max}(\theta_{Z_0=1} A = 0, Z = 2)]$
Always Having High Employment Regardless of Treatment	$\theta_{Z_1=2} > \text{Max}[\text{Max}(\theta_{Z_1=2} A = 1, Z = 0), \text{Max}(\theta_{Z_1=2} A = 1, Z = 1)]$ and $\theta_{Z_0=2} > \text{Max}[\text{Max}(\theta_{Z_0=2} A = 0, Z = 0), \text{Max}(\theta_{Z_0=2} A = 0, Z = 1)]$
Probably Having Any Level of Employment under LFA and Control	$\text{Max}[\text{Min}(\theta_{Z_a=z} A = a, Z = 0), \text{Min}(\theta_{Z_a=z} A = a, Z = 1), \text{Min}(\theta_{Z_a=z} A = a, Z = 2)]$ $\leq \theta_{Z_a=z} \leq$ $\text{Min}[\text{Max}(\theta_{Z_a=z} A = a, Z = 0), \text{Max}(\theta_{Z_a=z} A = a, Z = 1), \text{Max}(\theta_{Z_a=z} A = a, Z = 2)]$ for $z = 0, 1, 2$ and $a = 0, 1$ .